Models of Division

Equal Sharing Models

An *equal sharing* or *partitive* model of division is a situation where the dividend represents the number of objects and the divisor represents the number of parts (or groups) that the objects will be distributed among. For example, in the problem $12 \div 4$, the 12 stands for the number of objects and the 4 stands for the number of groups. The quotient is 3, which tells us the number of objects that will be placed into each group.

For example, if you have 12 cookies divided into 4 equal parts, there will be 3 cookies in each part. If you have 12 inches divides into 4 equal parts, there will be 3 inches in each part.

Repeated Subtraction Models

A *repeated subtraction* or *measurement* model of division is a situation where the dividend represents the number of objects and the divisor represents the size of each group. For example, in the problem $12 \div 4$, the 12 stands for the number of objects and the 4 stands for the size of each group. The quotient is 3, which tells us the number of groups we will be able to make.

For example, if you have 12 cookies to divide into bags of 4 cookies, you can make 3 bags. This approach to division is also especially useful for illustrating division involving fractions. For example, if you have a $7\frac{1}{2}$ foot pipe and you want to divide it into $\frac{3}{4}$ foot segments, you will get 10 pieces of pipe.

Division as the Inverse of Multiplication

Another key way to think of division is to think of it as the inverse operation for multiplication. This approach is especially helpful in situations where the multiplicative relationship is more natural to understand. For example, the area of a rectangle can be found by multiplying the length by the width. Suppose that you know the area is 30 cm^2 and that the width is 5 cm. What is the length of the rectangle? This problem can be solved by dividing 30 by 5 or by thinking "5 times what equals 30?" For another example, consider $50 \div 10\% = w$. It it is most natural to think of this as "50 is 10% of what?"

Distinguishing Equal Sharing and Repeated Subtraction Division Models

State whether an equal sharing or a repeated subtraction model for division more accurately reflects each of the following problem scenarios. Remember that in an equal sharing model, you are told the number of groups, while in a repeated subtraction model, you are told the size of each group.

1. I received a $50 gift card for a local deli shop. If it costs me about $5.87 for lunch each time I go there, how many free lunches can I get?
2. The federal government allocated $787,000,000,000 (787 billion dollars) for the economic stimulus bill. If the current population of the United States is 307,212,123, how much money does this represent per person?
3. Consumers in the United States use 380 billion plastic bags each year. When compressed, 50 plastic bags take up about 1 cubic foot. How many total cubic feet would these bags take up if they end up in landfills?
4. I am planning a bicycle trip to San Antonio, which is about 1,500 miles by the route I am planning to take. If I need to arrive within 30 days, how many miles must I average each day?
5. A package of thirty cookies has an equal number of cookies in three different rows. How many cookies are in each row?
6. There is a 9 foot separation between one floor of a building and the next floor up. A carpenter needs to build steps that are each $\frac{3}{4}$ of a foot high to connect the two levels. How many stairs are needed?
7. One package of gummy bears contains 180 bears. To do a class activity, each student needs 30 bears. How many students can do the activity using only one package?
8. Three roommates decide to split the $960 monthly rent for their apartment equally. How much must each of them pay per month?

Writing Division Scenarios

Try this yourself! Write two scenarios illustrating equal sharing, and two scenarios illustrating repeated subtraction. Challenge someone else to determine which is which and to answer your questions.

Activity One

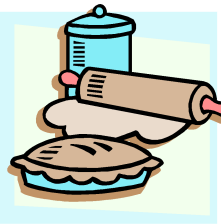
For each scenario, use pictures, equations, and a sentence to explain the situation. Use as many different equation forms as possible.

a) 7 children shared 18 pennies. How many pennies will each child have?

b) 31 cookies were given to 6 children to share fairly. How many cookies will each child have?

c) 8 children divide 27 pencils equally. How many should each child take?

Activity Two



A. There are 25 apples in the bag. I want to use 6 apples in each pie. How many pies can I make? Describe this situation with pictures, equations, and a sentence.

B. I want to put 3 plants in each flower pot. I have 29 plants. How many flower pots do I need? Describe this situation with pictures, equations, and a sentence.

How Many Stars?

A) There are 34 stars in the box below. How many groups of 2 do you think you will circle? _____

Explain your reasoning: _____

_____

_____

_____

_____

_____

Circle groups of 2.

How many groups did you circle? _____

Write as many equations as you can that describe this situation.

B) There are 56 stars in the box below. You will circle groups of 4. Use the partial quotients method to determine how many groups of 4 there will be.

How many groups of 4 do you think you will circle? _____

Explain your reasoning: _____

_____

_____

_____

_____

_____

Circle groups of 4.

How many groups did you circle? _____

Write as many equations as you can that describe this situation.

Activity One

Answer Key

For each scenario, use pictures, equations, and a sentence to explain the situation. Use as many different equation forms as possible.

a) 7 children shared 18 pennies. How many pennies will each child have?

$18 \div 7 = 2$ r4

$\frac{18}{7} = 2$ r4

$2 \times 7 + 4 = 18$

$18 - 2 - 2 - 2 - 2 - 2 - 2 - 2 = 4$

$18 - 7 - 7 = 4$

$2 + 2 + 2 + 2 + 2 + 2 + 2 + 4 = 18$

$7 + 7 + 4 = 18$

Each child will have 2 pennies with 4 pennies left over.

b) 31 cookies were given to 6 children to share fairly. How many cookies will each child have?

$31 \div 6 = 5$ r1

$\frac{31}{6} = 5$ r1

$6 \times 5 + 1 = 31$

$31 - 5 - 5 - 5 - 5 - 5 - 5 = 1$

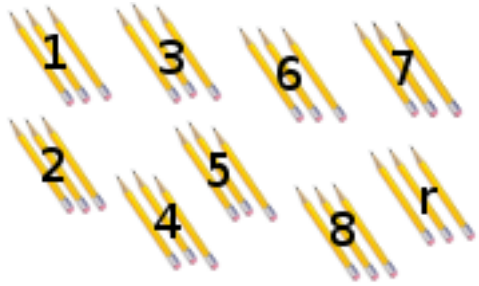
$31 - 6 - 6 - 6 - 6 - 6 = 1$

$5 + 5 + 5 + 5 + 5 + 5 + 1 = 31$

$6 + 6 + 6 + 6 + 6 + 1 = 31$

Each child will have 5 cookies with 1 cookie left over.

c) 8 children divide 27 pencils equally. How many should each child take?



$27 \div 8 = 3$ r3

$\frac{27}{8} = 3$ r3

$3 \times 8 + 3 = 27$

$27 - 3 - 3 - 3 - 3 - 3 - 3 - 3 - 3 = 3$

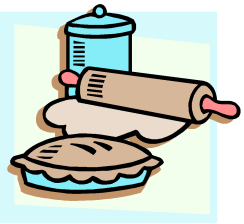
$27 - 8 - 8 - 8 = 3$

$3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 = 27$

$8 + 8 + 8 + 3 = 27$

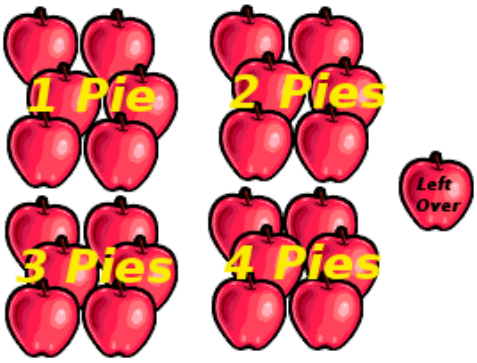
Each child should take 3 pencils and there will be 3 left over.

Activity Two



Answer Key

A. There are 25 apples in the bag. I want to use 6 apples in each pie. How many pies can I make? How many pies can I make? Describe this situation with pictures, equations, and a sentence.



You can make 4 pies with 1 apple left over.

$$\mathbf{25 \div 6 = 4\ r1}$$

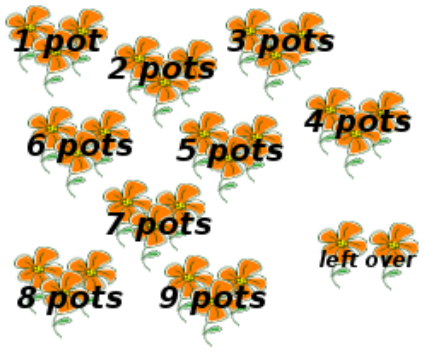
$$\mathbf{\frac{25}{6} = 4\ r1}$$

$$\mathbf{6 \times 4 + 1 = 25}$$

$$\mathbf{25 - 6 - 6 - 6 - 6 = 1}$$

$$\mathbf{6 + 6 + 6 + 6 + 1 = 25}$$

B. I want to put 3 plants in each flower pot. I have 29 plants. How many flower pots do I need? Describe this situation with pictures, equations, and a sentence.



$$29 \div 3 = 9 \text{ r}2$$

$$\frac{29}{3} = 9 \text{ r}2$$

$$3 \times 9 + 2 = 29$$

You need 9 pots with 2 plants left without a pot, or 10 pots with the last pot having only 2 plants.

$$29 - 3 - 3 - 3 - 3 - 3 - 3 - 3 - 3 - 3 = 2$$

$$3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 2 = 29$$

How Many Stars?

Answer Key

A) There are 34 stars in the first box. How many groups of 2 do you think you will circle? **16** Explain your reasoning: **Accept any reasonable answer: e.g. I think there will be more than 10 groups, because 10 groups of 2 would be 20 stars. I think there will be fewer than 20 groups, because 20 groups of 2 would be 40 stars.**

Circle groups of 2.

How many groups did you circle **17**

Write as many equations as you can that describe this situation.

$34 \div 2 = 17$ or $\frac{34}{2} = 17$

$2 \times 17 = 34$ or $17 \times 2 = 34$

$34 - 17 - 17 = 0$

$34 - 2 - 2 - 2 - 2 - 2 - 2 - 2 - 2 - 2 - 2 - 2 - 2 - 2 - 2 - 2 - 2 = 0$

$17 + 17 = 34$

$2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 = 34$

B) There are 56 stars in the second box. You will circle groups of 4. Use the partial quotients method to determine how many groups of 4 there will be.

Stars Left	Circled Groups	Stars Circled
56	10	40
16	4	16
0		

Total circled groups: $10 + 4 = 14$.

How many groups of 4 do you think you will circle? 14

Explain your reasoning:
You can make 14 groups of 4 out of 56; 10 groups of 4 is 40 and 4 more groups of 4 is 16 which makes 56.

Circle groups of 4.

How many groups did you circle? 14

Write as many equations as you can that describe this situation.

$56 \div 4 = 14$ or $\frac{56}{4} = 14$

$4 \times 14 = 56$ or $14 \times 4 = 56$

$56 - 10 \times 4 - 4 \times 4 = 0$