Demystifying Division

with Cookies and Coins

Students develop an understanding of division of whole numbers using equal sharing and repeated subtraction models to solve problems.

Grade Level: 3rd

Topics: whole number division, relationship between division and multiplication, division notation, meaning of remainders, problem solving, communicating and modeling mathematical ideas

Common Core State Standards: 3.OA.2, 3.OA.3, 3.OA.4, 4.OA.3

Indiana Math Standards: 3.C.2, 3.C.3, 3.C.4, 4.C.3

Goals:

- Students will be able to distinguish equal sharing and repeated subtraction models of division.
- Students will be able to write division equations using two representations:
  - $a \div b = c$
  - $\frac{a}{b} = c$
- Students will be able to correctly interpret remainders within the context of a given problem.
- Students will be able to write multiplication, subtraction, and addition equations which express ideas equivalent to a given division equation.
- Students will be able to communicate their mathematical thinking verbally, with models or pictures, and in writing.

Prerequisite Knowledge:

- Students should be able to count by tens.
- Students should have worked with multiplication models.

Teacher Materials:

- *The Doorbell Rang* by Pat Hutchins
- *One Hundred Hungry Ants* by Elinor J. Pinczes and Bonnie MacKain
- a large quantity of pennies or tokens (approximately 300)
- chart paper
- 5”× 8” index cards labeled with numbers of cookies and numbers of people (see Lesson 4)

Student Materials (needed for each student)**:**

- 24 tokens
- sheet of manila paper
- small cup or bowl
- student activity sheets:
  - *Activity One*
  - *Activity Two*
  - *How Many Stars?*
- 5"× 8" index cards
- *OPTIONAL:*
  - cookie cereal (see Lesson 1)
  - small white boards and dry erase markers
  - place mats
  - sticky notes

Preparation Time: 1 hour

Activity Time: 8 lessons

References:

1. Marilyn Burns, *Lessons for Introducing Division: Grades 3–4*
2. Van de Walle and Lovin, *Teaching Student-Centered Mathematics Grades 5–8, Vol. 3*

Additional Resources:

- Online Activities at *illuminations.nctm.org*
  - *Coin Box* – using arrays to exchange coins
  - *Factorize* – dividing numbers into factors and arrays
  - *Product Game* – building multiplication skills
  - *Times Table* – interactive multiplication table
- Elementary/Arithmetic Lessons at *www.nsa.gov*
  - *Defining Division*
  - *March of the Dividing Ant*

Concept Review For Teachers

The lessons in this module expose students to the different models for division which the Common Core State Standards (and Indiana State Standards) ask 3rd and 4th graders to learn. There are really three different models for division. In educational literature these are often called the partitive model of division, the measurement model of division, and division as the inverse of multiplication. The specific names given do not matter very much. Indeed the standards which relate most closely to this idea do not use this language.

Common Core State Standard 3.OA.2 Interpret whole-number quotients of whole numbers, e.g., interpret $56 \div 8$ as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as* $56 \div 8$.

Indiana Math Standard 3.C.3 Represent the concept of division of whole numbers with the following models: partitioning, sharing, and an inverse of multiplication. Understand the properties of 0 and 1 in division.

It is helpful for students to have an awareness that there are different models, and to be able to refer to these ideas. However, the specific terms are not at all important, and so teachers and students should feel free to make up and use whatever terminology seems natural to them. The lessons below suggest calling the partitive model "equal sharing", and the measurement model "repeated subtraction".

The "Models of Division" handout reviews the two interpretations of division and gives an opportunity to practice identifying and writing division scenarios.

Lesson Plans

Lesson 1: Students work with an equal sharing model of division.

Students are given 12 tokens (or pieces of cookie cereal) and a mat or manila paper. The teacher begins reading *The Doorbell Rang*. After the first two pages, the teacher asks the students to move the tokens around on the mat in order to model this situation. The teacher then writes "2 children shared 12 cookies" along with the equations $12 \div 2 = 6$ and $\frac{12}{2} = 6$.

Next, the teacher writes, "Each of the 2 children had 6 cookies," and invites students to think of an equation that would go with this idea. Students should be able to suggest that this would be $2 \times 6 = 12$ or $6 \times 2 = 12$. We could also write $6 + 6 = 12$, which would describe how the

6 cookies in one group and the 6 cookies in the other group total 12 cookies. We could write $2+2+2+2+2+2=12$ to describe how we doled out the cookies – we gave out two cookies in the first round, and continued like that until the 12 cookies were used up. We could also write $12-6-6=0$, which shows that if we start with 12 and give away two groups of 6, all of the cookies are used up. By the same token, we could write $12-2-2-2-2-2-2=0$. For each equation, we need to be able to explain how it relates to the given situation.

Ask the students to model with tokens and write equations each time more children in the story arrive to share the cookies.

The teacher asks students to put 18 tokens (or pieces of cookie cereal) on the mat. Students work with a partner to discuss and model three children sharing 18 cookies. Students move the tokens on the mat to show how the cookies would be shared. Using paper and pencil or white boards, students should write the two forms of division equations, as well as multiplication, subtraction, and addition problems.

After the story is over, students should try the following problems on their own, talking as a full group after each one.

- 2 children share 18 cookies
- 18 cookies are shared by 6 children
- 9 children divide 18 cookies among them

Lesson 2: Students find numbers that make good fair division problems.

The teacher should remind students that in the previous lesson, we found that when 18 cookies are shared by 6 children, each child gets 3 cookies. This situation can be represented by several different kinds of equations involving division (in both forms), multiplication, subtraction, and addition. Ask students to help make a list of these equations.

The students will now use tokens to find good numbers for fair division problems. Each student should choose between 15 and 30 tokens and make up a division scenario involving that many cookies. They should make a list of the division scenarios they discover where a certain number of cookies can be shared equally by some number of children with no cookies left over. For each scenario, they should write as many equations as they can think of which show the situation.

After five minutes of independent work, ask a few students to share their ideas and correct any misunderstandings about how the representations work. Allow students to have more time for independent work. After another five or ten minutes, ask students to get into pairs or small groups to check each other's equation ideas to decide if they are correct and to help think of more equation

ideas.

Lesson 3: Students work with remainders.

Each student is given 24 tokens and a mat. The teacher writes the following sentence on a chart: "13 cookies are shared by 5 people." Students work to determine how many cookies each of the 5 people will have. Students should share their ideas with a partner and then with the class.

Students should notice that there are "leftovers". The teacher should ask students what should be done with the 3 cookies left over and how this could be shown with a division equation. The teacher can then write $13 \div 5 = 2$ r 3, as well as the fraction representation of this problem. Have students discuss the meaning of this remainder. The teacher should then write, "Each of the 5 children had 2 cookies and there are 3 cookies left over." Invite students to try to show this with multiplication ($5 \times 2 = 10$ explains how each of the children had two cookies, but what happens to the leftovers? The equation $5 \times 2 + 3 = 13$ includes the leftovers also). We could also write many equations involving addition and subtraction equations that match the situation.

Using the tokens, students should complete *Activity One*.

The teacher should hand out 5"× 8" cards to each student. Each student should write their own problem on the lined side of the card. On the back, students should provide a solution to the problem. This solution should include equations, pictures and a written explanation.

Lesson 4: Review

The teacher will have 5"× 8" cards labeled "11 cookies," "12 cookies," etc. up to "30 cookies" for half the students in class and 5"× 8" cards labeled "2 children," "3 children," etc. up to "10 children" for the other half of the students. There should be a designated area for students to write division problems (chalkboard, chart on the wall, white board, etc). The teacher will explain that students are going to be playing a game. The object of the game is for the class to write and solve as many division problems as possible in 5 minutes. One player with a cookie card must find a player with a children card. As a team, they must think of the division problem and its solution using mental math, tokens, or pictures. They must write the division equation in the designated area. For example, 11 cookies and 3 children would be $11 \div 3 = 3$ r2.

When the game is done, the class should review each division problem.

Students should then exchange the 5"× 8" card problems written during the previous lesson with a partner. Partners exchange problems and work to solve them. Each person should compare their result with the solution on the back of the card. Then, have partners exchange with other teams.

Lesson 5: Students use a repeated subtraction model of division.

The teacher should then read aloud from *One Hundred Hungry Ants* and students will use white boards to write numerical equations of multiplication and division. For example: There are 100 ants. If they place 25 ants in each line, how many lines would there be? This could be expressed by the division problem $100 \div 25 = 4$. The way we think about division changes depending on whether we know the number of groups or whether we know the number in each group. When we know the number of groups, we can dole out the objects into groups until they are all placed. If we know the number of objects in each group instead, we form one group at a time until the objects are used up. The first idea is called the "equal sharing" approach to division. The second idea is called the "repeated subtraction" approach to division. During the earlier lessons, we worked with the "equal sharing" approach, and so now we will switch and practice working with the "repeated subtraction" approach.

The teacher shows a bowl containing 35 pennies to the class (or uses 35 magnetic pennies). We want to buy stickers that cost 7 cents each. How many could we buy? This situation can be represented as $35 \div 7 =?$. Ask the students to make some guesses about the answer. The teacher should then make piles of 7 in full view of the class. Each time a group of 7 is drawn off, the teacher subtracts 7 from 35 and finds the current total. When the last group of 7 is set aside, the last computation should end in a zero. Since we were able to subtract 7 five times the answer is $35 \div 7 = 5$. In other words, since we were able to make five piles with 7 cents in each one, we know that we can buy 5 stickers.

We can use the same approach in situations where there is a remainder. For example, if we have 23 pennies, and we want to buy stickers that cost 5 cents each, we can ask the question $23 \div 5 =?$. This time, we can make four groups of 5 pennies, but we have 3 left over. This means that we can buy four stickers and we will have three pennies remaining. Model this problem by repeatedly subtracting 5 from 23, showing that we are left with 3 at the end of the process, and that we were able to subtract 5 four times.

Lesson 6: Students will translate verbal problems into division problems and solve.

As a review of Lesson 5, the teacher should group students in pairs. Remind students that we are thinking about division as repeated subtraction instead of using equal sharing. Ask students to figure out how many 8-cent stickers they could buy with 29 cents. After allowing some time for independent work, ask students to show how to write the related division problem and repeated subtraction problem.

Students should work with a partner to complete *Activity Two*. Afterward, the class should discuss

strategies and solutions. Focus especially on how they might want to think about the remainders in each case. In the case of the pies, how many pies can they make? If they have a certain number of flowers and want to plant them all, putting no more than three flowers in each pot, how many pots would they need? If they only want to make complete pots of three flowers, and will give away the extra flowers, how many pots would they need then? It is important to think about the context to know what answer to give.

Lesson 7: Students write division scenarios

The teacher should write the division equation $\frac{12}{4} =?$, and ask a student to come up with a problem scenario that fits this question. Write the scenario on the board so that everyone can see it. Ask the class to first consider whether it really fits the question. If it does, ask them whether the scenario is an example of equal sharing or repeated subtraction. Check whether that seems to be true by using objects to model how we would think about the situation. Ask someone else to suggest another scenario which uses the opposite approach. Write that one on the board and ask students to think about whether it matches the division question, and whether it is of the opposite type. Use objects to model how we would think about that situation.

Ask students to work in pairs or small groups to write four division scenarios.

1. Write one scenario which uses the "equal groups" approach which does not have a remainder.
2. Write one scenario which uses the "equal groups" approach which does have a remainder.
3. Write one scenario which uses the "repeated subtraction" approach which does not have a remainder.
4. Write one scenario which uses the "repeated subtraction" approach which does have a remainder.

If time allows, students can exchange problems with another group and solve them.

Lesson 8: How Many Stars?

The teacher should hand out the activity sheet *How Many Stars?*. Students should first estimate how many groups of 2 they will circle, and then work independently to circle groups of two. They should write as many true equations as they can which fit the situation.

Next, the teacher introduces the idea of *partial quotients*: "In the first picture, there are 34 stars and you made groups of 2. Did you know that there were at least 10 groups of 2? Why is it easy to know that is true?" Ask students how many they would like to circle at the next jump and write the following in words. "We started with 34 stars, if we circle 10 groups of two, that would take care of 20 stars, leaving 14 undone." What number of groups of 2 should we take care of next? Someone

suggests circling 4 groups. “If we circle 4 groups of two, that takes care of 8 more stars, leaving 6.” How many groups of 2 can we circle now? “Finally, we circle 3 groups of two”. Go back through the narrative and underline or circle all of the numbers of groups: 10, 4, and 3. “Altogether, we made $10 + 4 + 3 = 17$ groups of two out of the 34 stars.”

Point out to the students that there are many ways they might organize and keep track of this process. They could use a table, a number line, equations, or any other way that makes sense to them. Try to avoid giving them a particular way to do this or any kind of rule for deciding what jumps to make.

Next, students should try to use partial quotients to figure out how many circle groups they will have in the second picture. After they use partial quotients to make their best guess, they should actually circle groups of 4 to see if they were close.

Give each student manila paper and markers. Students will create their own star (or dot) pictures to group and will practice the partial quotients strategy. Allow students to share their strategies and their methods for keeping track of the number of groups.

Models of Division

Equal Sharing Models

An *equal sharing* or *partitive* model of division is a situation where the dividend represents the number of objects and the divisor represents the number of parts (or groups) that the objects will be distributed among. For example, in the problem $12 \div 4$, the 12 stands for the number of objects and the 4 stands for the number of groups. The quotient is 3, which tells us the number of objects that will be placed into each group.

For example, if you have 12 cookies divided into 4 equal parts, there will be 3 cookies in each part. If you have 12 inches divides into 4 equal parts, there will be 3 inches in each part.

Repeated Subtraction Models

A *repeated subtraction* or *measurement* model of division is a situation where the dividend represents the number of objects and the divisor represents the size of each group. For example, in the problem $12 \div 4$, the 12 stands for the number of objects and the 4 stands for the size of each group. The quotient is 3, which tells us the number of groups we will be able to make.

For example, if you have 12 cookies to divide into bags of 4 cookies, you can make 3 bags. This approach to division is also especially useful for illustrating division involving fractions. For example, if you have a $7\frac{1}{2}$ foot pipe and you want to divide it into $\frac{3}{4}$ foot segments, you will get 10 pieces of pipe.

Division as the Inverse of Multiplication

Another key way to think of division is to think of it as the inverse operation for multiplication. This approach is especially helpful in situations where the multiplicative relationship is more natural to understand. For example, the area of a rectangle can be found by multiplying the length by the width. Suppose that you know the area is 30 cm^2 and that the width is 5 cm. What is the length of the rectangle? This problem can be solved by dividing 30 by 5 or by thinking "5 times what equals 30?" For another example, consider $50 \div 10\% = w$. It it is most natural to think of this as "50 is 10% of what?"

Distinguishing Equal Sharing and Repeated Subtraction Division Models

State whether an equal sharing or a repeated subtraction model for division more accurately reflects each of the following problem scenarios. Remember that in an equal sharing model, you are told the number of groups, while in a repeated subtraction model, you are told the size of each group.

1. I received a $50 gift card for a local deli shop. If it costs me about $5.87 for lunch each time I go there, how many free lunches can I get?
2. The federal government allocated $787,000,000,000 (787 billion dollars) for the economic stimulus bill. If the current population of the United States is 307,212,123, how much money does this represent per person?
3. Consumers in the United States use 380 billion plastic bags each year. When compressed, 50 plastic bags take up about 1 cubic foot. How many total cubic feet would these bags take up if they end up in landfills?
4. I am planning a bicycle trip to San Antonio, which is about 1,500 miles by the route I am planning to take. If I need to arrive within 30 days, how many miles must I average each day?
5. A package of thirty cookies has an equal number of cookies in three different rows. How many cookies are in each row?
6. There is a 9 foot separation between one floor of a building and the next floor up. A carpenter needs to build steps that are each $\frac{3}{4}$ of a foot high to connect the two levels. How many stairs are needed?
7. One package of gummy bears contains 180 bears. To do a class activity, each student needs 30 bears. How many students can do the activity using only one package?
8. Three roommates decide to split the $960 monthly rent for their apartment equally. How much must each of them pay per month?

Writing Division Scenarios

Try this yourself! Write two scenarios illustrating equal sharing, and two scenarios illustrating repeated subtraction. Challenge someone else to determine which is which and to answer your questions.

Activity One

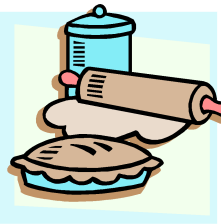
For each scenario, use pictures, equations, and a sentence to explain the situation. Use as many different equation forms as possible.

a) 7 children shared 18 pennies. How many pennies will each child have?

b) 31 cookies were given to 6 children to share fairly. How many cookies will each child have?

c) 8 children divide 27 pencils equally. How many should each child take?

Activity Two



A. There are 25 apples in the bag. I want to use 6 apples in each pie. How many pies can I make? Describe this situation with pictures, equations, and a sentence.

B. I want to put 3 plants in each flower pot. I have 29 plants. How many flower pots do I need? Describe this situation with pictures, equations, and a sentence.

How Many Stars?

A) There are 34 stars in the box below. How many groups of 2 do you think you will circle? _____

Explain your reasoning: _____

_____

_____

_____

_____

_____

Circle groups of 2.

How many groups did you circle? _____

Write as many equations as you can that describe this situation.

B) There are 56 stars in the box below. You will circle groups of 4. Use the partial quotients method to determine how many groups of 4 there will be.

How many groups of 4 do you think you will circle? _____

Explain your reasoning: _____

_____

_____

_____

_____

_____

Circle groups of 4.

How many groups did you circle? _____

Write as many equations as you can that describe this situation.

Activity One

Answer Key

For each scenario, use pictures, equations, and a sentence to explain the situation. Use as many different equation forms as possible.

a) 7 children shared 18 pennies. How many pennies will each child have?

$18 \div 7 = 2$ r4

$\frac{18}{7} = 2$ r4

$2 \times 7 + 4 = 18$

$18 - 2 - 2 - 2 - 2 - 2 - 2 - 2 = 4$

$18 - 7 - 7 = 4$

$2 + 2 + 2 + 2 + 2 + 2 + 2 + 4 = 18$

$7 + 7 + 4 = 18$

Each child will have 2 pennies with 4 pennies left over.

b) 31 cookies were given to 6 children to share fairly. How many cookies will each child have?

$31 \div 6 = 5$ r1

$\frac{31}{6} = 5$ r1

$6 \times 5 + 1 = 31$

$31 - 5 - 5 - 5 - 5 - 5 - 5 = 1$

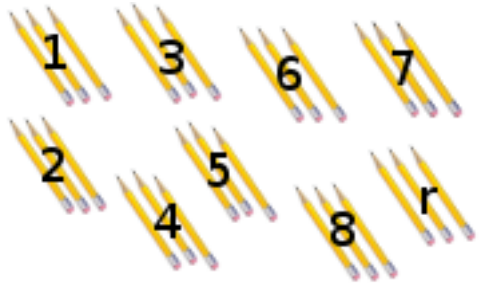
$31 - 6 - 6 - 6 - 6 - 6 = 1$

$5 + 5 + 5 + 5 + 5 + 5 + 1 = 31$

$6 + 6 + 6 + 6 + 6 + 1 = 31$

Each child will have 5 cookies with 1 cookie left over.

c) 8 children divide 27 pencils equally. How many should each child take?



$27 \div 8 = 3$ r3

$\frac{27}{8} = 3$ r3

$3 \times 8 + 3 = 27$

$27 - 3 - 3 - 3 - 3 - 3 - 3 - 3 - 3 = 3$

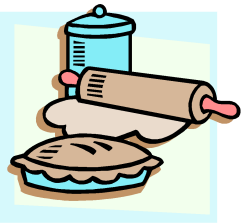
$27 - 8 - 8 - 8 = 3$

$3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 = 27$

$8 + 8 + 8 + 3 = 27$

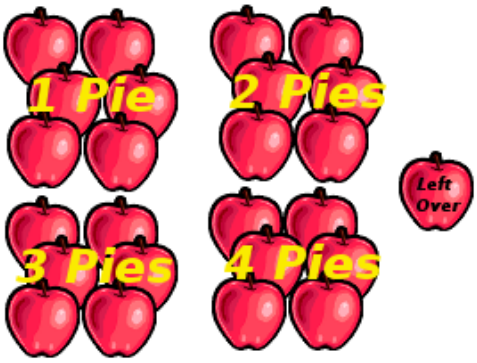
Each child should take 3 pencils and there will be 3 left over.

Activity Two



Answer Key

A. There are 25 apples in the bag. I want to use 6 apples in each pie. How many pies can I make? How many pies can I make? Describe this situation with pictures, equations, and a sentence.



You can make 4 pies with 1 apple left over.

$$\mathbf{25 \div 6 = 4\ r1}$$

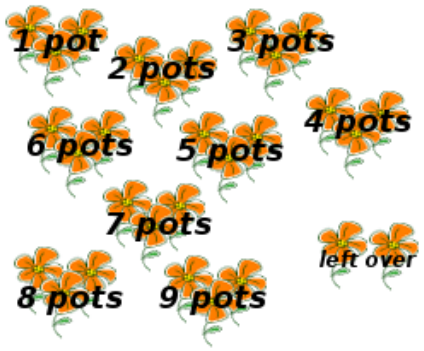
$$\mathbf{\frac{25}{6} = 4\ r1}$$

$$\mathbf{6 \times 4 + 1 = 25}$$

$$\mathbf{25 - 6 - 6 - 6 - 6 = 1}$$

$$\mathbf{6 + 6 + 6 + 6 + 1 = 25}$$

B. I want to put 3 plants in each flower pot. I have 29 plants. How many flower pots do I need? Describe this situation with pictures, equations, and a sentence.



$$29 \div 3 = 9 \text{ r}2$$

$$\frac{29}{3} = 9 \text{ r}2$$

$$3 \times 9 + 2 = 29$$

You need 9 pots with 2 plants left without a pot, or 10 pots with the last pot having only 2 plants.

$$29 - 3 - 3 - 3 - 3 - 3 - 3 - 3 - 3 - 3 = 2$$

$$3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 2 = 29$$

How Many Stars?

Answer Key

A) There are 34 stars in the first box. How many groups of 2 do you think you will circle? **16** Explain your reasoning: **Accept any reasonable answer: e.g. I think there will be more than 10 groups, because 10 groups of 2 would be 20 stars. I think there will be fewer than 20 groups, because 20 groups of 2 would be 40 stars.**

Circle groups of 2.

How many groups did you circle **17**

Write as many equations as you can that describe this situation.

$34 \div 2 = 17$ or $\frac{34}{2} = 17$

$2 \times 17 = 34$ or $17 \times 2 = 34$

$34 - 17 - 17 = 0$

$34 - 2 - 2 - 2 - 2 - 2 - 2 - 2 - 2 - 2 - 2 - 2 - 2 - 2 - 2 - 2 - 2 = 0$

$17 + 17 = 34$

$2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 = 34$

B) There are 56 stars in the second box. You will circle groups of 4. Use the partial quotients method to determine how many groups of 4 there will be.

Stars Left	Circled Groups	Stars Circled
56	10	40
16	4	16
0		

Total circled groups: $10 + 4 = 14$.

How many groups of 4 do you think you will circle? 14

Explain your reasoning:
You can make 14 groups of 4 out of 56; 10 groups of 4 is 40 and 4 more groups of 4 is 16 which makes 56.

Circle groups of 4.

How many groups did you circle? 14

Write as many equations as you can that describe this situation.

$56 \div 4 = 14$ or $\frac{56}{4} = 14$

$4 \times 14 = 56$ or $14 \times 4 = 56$

$56 - 10 \times 4 - 4 \times 4 = 0$