# Do You Want to Know a Secret?

Students explore function rules and linear relationships through input and output tables and tile patterns.

**Grade Level**: 5th – 8th

**Topics**: patterns, input/output tables, functions, variable expressions, dependent and indipendent variables, ordered pairs, graphing linear equations

**Goals**:
- Students will be able to describe a pattern using words.
- Students will be able to describe a pattern using variables.
- Students will be able to create multiple representations of a linear function: equation, table, graph.
- Students will be introduced to rate of change and slope.

**Prerequisite Knowledge**:
- Students are able to write and evaluate variable expressions.
- Students are able to graph coordinates in the Cartesian Plane.

| **Math Standards**: | | | | |
|---|---|---|---|---|
| | 5.OA.1 | 6.EE.2 | 7.EE.1 | 8.EE.5 |
| | 5.OA.2 | 6.EE.4 | 7.EE.2 | 8.F.1 |
| | 5.OA.3 | 6.EE.5 | 7.EE.3 | 8.F.3 |
| | | 6.EE.6 | | 8.F.4 |
| | | | | 8.F.5 |

**Materials**:
- square tiles (approximately 20 per student)
- large chart paper to display in the room
- grid paper
- Cartesian plane sheets
- rulers
- standard calculators
- student activity sheets
  - “Function Game”
  - “Secret Calculators”
  - “Tile Patterns Activity 1”
  - “Tile Patterns Activity 2”
- 3'' x 5'' cards
- TI – 73 graphing calculators (for extension)

**Preparation Time**: 30 minutes

**Activity Time**: 5 – 8 lessons

**Primary Sources**:

1. West, Kelsch, Sowers, “Algebraic thinking: Becoming Proficient with Patterns”, NCTM Conference, Indianapolis, April, 2011
(Tile patterns from this presentation are incorporated in three lessons.)

2. Rubenstein, “The Function Game”, *Mathematics Teaching in the Middle School,* Nov.-Dec., 1996.
(Suggestions from this article are incorporated in the opening lesson.)

**References**:

1. Van de Walle and Lovin, *Teaching Student-Centered Mathematics Grades 5 – 8,* vol.3, Pearson, 2009.

# Lesson Plans

**Lesson One: Students play a game to become familiar with function rules.**

Prior to beginning this lesson, have several 3'' x 5'' cards prepared with various function rules. Suggestions are: add 3, subtract 7, multiply by 3, double a number and add 1, divide by 2, multiply by 3 and add 2, add 10, multiply by 5.

Have chart paper prepared with a table as shown below:

| Input | Output |
| --- | --- |
| | |

Tell students that you have a secret that you want them to try to guess. They will tell you a number, you will do something to that number, as determined by the card you have chosen, and they have to try to guess what it is you did. Hand out the “Function Game” activity sheet and calculators, so students can write down the numbers as they try to determine the secret rule that is written on the card. As students provide input numbers, write them in the table along with the output number.

When a student thinks he or she knows the secret, invite them to write it down and state “I know your secret!”. You will give them an input number and if they can tell you the correct output number, you will say, “You know the secret!”. Continue until the majority of the students say they know what is happening to the input number. Have students who know the secret prove that they are correct by describing how the output number comes from the input number.

Have students share strategies that helped them. For example, students may notice that an input of 0 is helpful to decide if a number is multiplied or divided by another number. Also, encourage students to discuss why more than one input is necessary to decide what is happening to the number. It may be necessary to provide an example for them such as an input of 2, output of 4 – the rule could be double a number, it could be add 2 to a number, it could be multiply by 3 and subtract 2, etc.

Hand out 3'' x 5'' cards to students. Have students work with a partner to design their own secret rule(s). Each pair should have a calculator and a sheet of chart paper. Each team will create an input/output table on the chart paper for display. The secret rule is written on the 3 x 5 card and hidden behind the chart paper when it is either laid on the desk or hung on the wall. When teams are done, they will travel the room to try to learn all the secrets of their classmates.

At the conclusion of class, ask students what was the *variable* as they look at the input/output tables on the chart. Lead them to understand that both the input and output tables are variable since the numbers can change.

**Lesson Two: Students write function rules in words and symbols.**

Post the chart from Lesson One and ask students about each rule for the input/output tables. Ask students to think about how the rule could be written as a complete sentence. For example, if the first table’s rule is to “add 5”, the sentence would be “Add 5 to the input number to find an output number.” Do this for three or four examples.

Refer back to the idea of variables from the end of lesson one. If input numbers and output numbers are variable, ask students what letters could be used to represent them. Allow any reasonable variables and write those letters above the input and output columns in each table. Invite students to work in pairs to try to write the rules using variables and numerals in place of the words. Have students share their results. In the example of paragraph one, students could use $n$ for input and $p$ for output and their rule could be $5 + n = p$.

Hand out the “Secret Calculators” activity sheet. Have chart paper prepared with input/output tables. Prepare the following 3'' x 5'' cards with these rules listed: add 8 to a number, multiply a number by 2 and add 5, subtract 2 from a number, add 3 to a number and then multiply by 2 (students will probably guess double a number and add 6, this would be an opportunity to discuss *equivalent expressions*). Each student should also have a calculator in order to test their guesses. Once again, if a student says, “I know the secret,” give them an input number and have them provide the output.

Work with students to write each “secret” in words. It is important that students create a complete sentence in order to more easily translate it into algebraic symbols. For number one, students would write, “Add 8 to an input number to create an output number.” Use the variables *x* and *y* for the input and output variables. This will be helpful for later work with graphing. For number one, students could then translate from words to the symbolic equation of $8 + x = y$. Students should also discuss whether or not $y = 8 + x$ *and* $x + 8 = y$ are the same. Replace the variables with several input/output numbers to show that the equation is a true representation of the pattern. In number one, if the input was 186 and the output was 194, then $8 + 186 = 194$ must be true.

Have students work with a partner to complete the activity sheet. Students should use the variable *x* for input and the variable *y* for output. Students are given the secret in words and must create an input/output table and the secret in symbols. Have four charts posted around the room. As each team completes the table, have them add their input/output numbers for each secret. Bring the class together and have students verify each other’s results and discuss the equations written.

Hand out 3'' x 5'' cards and have each student work independently to create a secret in words and symbols, as well as an input/output table.

**Lesson Three: Students use tile patterns to observe relationships and write rules using algebraic notation.**

As a review of Lesson Two, group students in teams of three or four. Have students share the work they did on the 3'' x 5'' cards, and have their results verified by the team. All symbolic “secrets” or equations should be written on the board. This may provide opportunities to discuss equivalent equations. Also, have students volunteer to use each equation to give a value for *x* and *y* that makes it true. Write each solution as an ordered pair, $(x,y)$.

Hand out colored square tiles to teams of two or three students. Each team should have twenty tiles. Each team should be given “Tile Patterns Activity 1.” Students will be looking at the first three pictures of a tile pattern and will construct the next two patterns. Students should discuss how the pattern is changing and use colors to denote the part of the pattern that is *constant* and the part of the pattern that is changing or *variable*. Students sketch models of the next two pictures and describe how the ninth pattern would look.

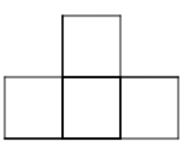
After allowing time for exploration, have teams share their results. More than one answer is possible (see teacher notes). Have students write the pattern number and amount of tiles as an input/output table as shown below:

| **Pattern Number (n)** | **Amount of Tiles (a)** |
|---|---|
| 1 | 3 |
| 2 | 4 |
| 3 | 5 |
| n | n+2 or 3+(n–1) or 1+(n+1) or any other equivalent statement |

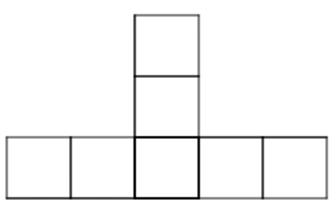
Have partners try the second pattern with their tiles and with sketching. Students should compare their rules with one another to determine if anyone has equivalent rules. Have students create the input/output table for the pattern number and amount of tiles.

**Lesson Four: Students sketch patterns on grid paper to observe relationships and write rules using algebraic notation.**

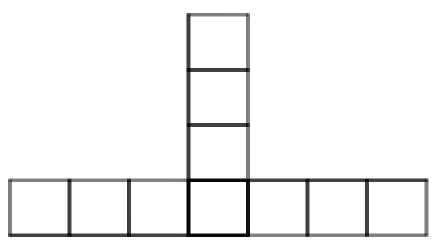
As a review of Lesson Three, have the following pattern on chart paper for students to see:



*Pattern 1*

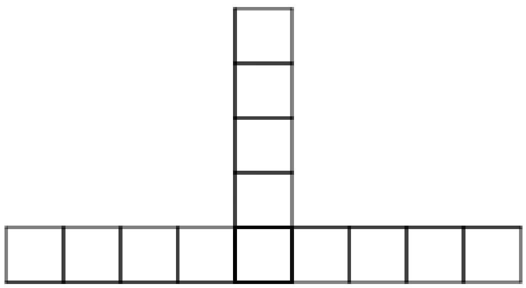


*Pattern 2*

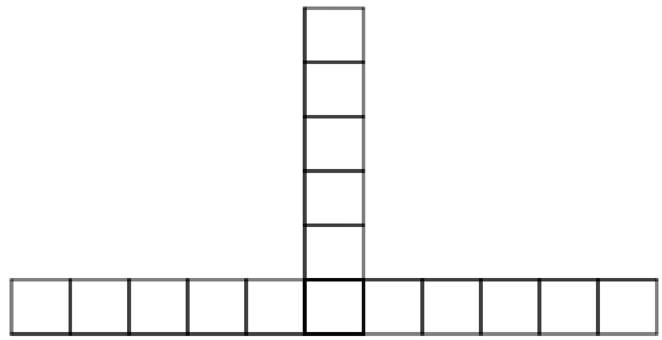


*Pattern 3*

Provide grid paper for students. Have students copy the three patterns onto grid paper. Students should shade the tile(s) they think remain constant. Students should compare their results with classmates. Have students explain their reasoning and explain how they think the pattern is changing. Ask, “What would the next two patterns be?” Have students draw them on their paper and then compare results with classmates.



*Pattern 4*



*Pattern 5*

Have the table below written on the chart paper. Have students help to complete the table for each pattern. After completing the first four patterns, have students discuss their observations. Students should think/pair/share to finish the *n* row.

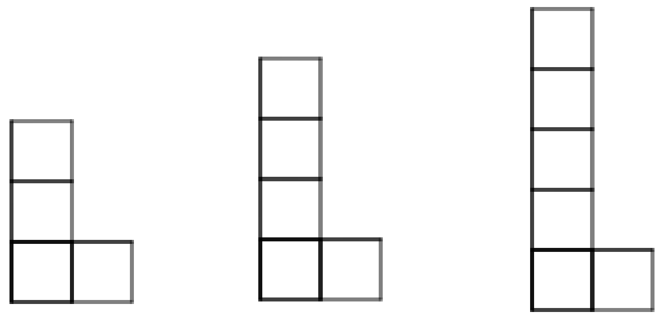
| **Pattern Number** | **Constant** | **Variable** | **Total Squares** |
|---|---|---|---|
| 1 | 1 | 3 | 4 |
| 2 | 1 | 6 | 7 |
| 3 | 1 | 9 | 10 |
| 4 | 1 | 12 | 13 |
| 5 | 1 | 15 | 16 |
| n | 1 | 3n | 1+3n or 3n+1 |

Have students predict the sixth pattern using the rule and then draw the pattern and check the results. Ask the question, “Which pattern number would have 151 squares?” Students should think/pair/share a strategy for determining the answer. Using algebra, students could write $3n+1 = 151$ and solve the equation. The pattern number is 50.

Hand out “Tile Patterns Activity 2”. Students should work in teams of two or three to discover the constant and the variable for the pattern and complete the chart.

**Lesson Five: Students create their own tile patterns and describe the patterns in words and with an algebraic rule.**

As a review of yesterday's work, have the following table and patterns on chart paper:



*Pattern 1* *Pattern 2* *Pattern 3*

| Input | Output |
| --- | --- |
| | |

Have students help to complete the input (pattern number) and output (amount of tiles) table and to explore the patterns. Ask students, "What is the rule?" Each picture has one more tile than the previous picture (iterative pattern) and the total number of tiles is the pattern number plus three. The rule for the *nth* picture would be $n + 3$ which is simplified from $(n + 1) + 2$.

Each student will be designing their own pattern using a letter in their first or last name. Provide grid paper and tiles for each student. Students must draw the first three stages of the pattern using a rule that they create. On separate paper, students create an answer key that includes: a) the next two patterns, b) pattern ***n*** written in words and as an algebraic rule, c) a sketch which shows which parts of the pattern are variable and which parts are constant, and d) an input and output table.

Students should exchange patterns with their classmates and try to determine the rules and then check their results using the answer keys.

**Lesson Six: Students graph function rules and explore linear relationships.**

Each student will need a copy of the Cartesian plane and a ruler. Students should also have copies of the "Secret Calculators" student activity from Lesson two. Ask students to predict what they will see if they graph the ordered pairs from each input/output table. Work with students to graph the coordinates from each rule. It may be helpful to use a different color for each rule. Ask students what they observe (the points are in a straight line). Have students draw lines through the points and ask them what they think the line represents. Lead students to understand that the line represents all the possible input/output values for the rule. Have students find other *solutions* for the input/output table by using the *graph* of the equation or rule. Have students test the solutions using the equation. For example, for *y = 5x + 1* students could select (0,1) and 5(0) + 1 = 1 is true. The point (-1, -4) is also on the line and 5(-1) + 1 = -4.

Have students work with a partner to graph the input/output tables for the patterns that were explored in lessons three and four. Students should think/pair/share why they think all of these points on the graph line up.

As a class, discuss why these patterns are *linear.* Lead students to understand that if each new pattern is increasing in the same way, the relationship is linear. Introduce the idea of a *steady rate of change.* For each pattern studied, the number of tiles increased at the same *rate.*

Students in seventh and eighth grade should also discuss *domain* and *range.* In the case of the calculator rules from lesson two, the domain and range are all real numbers. In the case of the tile patterns, the domain and range are only positive whole numbers.

**Lesson Seven: Students are introduced to slope as a rate of change.**

Post the chart and table from Lesson Four with the rule: ***a = 3n + 1.***

Have students discuss how each picture changed. Emphasize that as the pattern number increased by 1, the number of tiles increased by 3. The ratio is 3:1 or 3. This ratio is maintained even when you move from pattern number 1 to pattern number 4, which would be a ratio of 9:3 or 3. Introduce the idea of slope which compares the change in y (the output) to the change in x (the input).

Have students look at the graphs from the “Secret Calculators” activity sheet created in Lesson Six and the equations written for each rule. For example with the rule *y = 5x + 1,* when an input number increases by 1, the output increases by 5. If the input increases by 3, the output increases by 15. The ratio of *y* to *x*, or slope, is always 5 to 1. Students should be able to see this ratio on the graphs as well.

How can knowing slope or rate of change help to unlock secrets? Put the following input/output table on a chart:

| **Input (x)** | **Output (y)** |
|---|---|
| 0 | 6 |
| 1 | 11 |
| 4 | 26 |
| 10 | 56 |

Students should work with a partner to try to determine the secret rule for the input/output table. After students have worked and shared ideas, have them look at the points (1,11) and (4,26). How did the input change? (It increased by 3.) How did the output change? (It increased by 15.) What is the ratio of *y* to *x*? (5 to 1 or 5.)

Just as the table showed with tile patterns, this ratio becomes a multiplier of the input number. What would have to be added to 4 x 5 to equal 26? (6). Will the rule $y = 5x + 6$ work? Have students test every input number.

Each pair of students should now create an input/output table based on a rule written in the form $y = mx + b$ where *m* is the multiplier (i.e. slope) and *b* is the added constant.* These tables should be written on sheets of manila paper and the secret rule or equation should be written on a 3'' x 5'' card and placed underneath (or behind, if hanging) the poster. Students travel from poster to poster and work with their partner to determine the multiplier and the constant and to write the rule as an equation. Encourage students to test each input number to verify their results.

Discuss the experience as a class. Did finding the slope make finding the secret easier?

* 7th and 8th grade students could use negative numbers for multipliers.

**Extension: Students use graphing calculators to explore various slopes and y-intercepts and the form of a linear equation y = mx+b.**

Students will need to use a graphing calculator for this lesson. A TI-73 is adequate for the work. Students will need to have blank Cartesian planes to sketch what they view on the calculator screen. The goal is to have students notice how the slope or multiplier affects the graph and how the constant (y-intercept) affects the graph.

Students should do three or four equations on the same screen in order to compare graphs. It is best to enter them one at a time. Some suggested equations are listed below:

| Set 1: | Set 2: | Set 3: | Set 4: |
|---|---|---|---|
| $y = 2x$ | $y = 2x + 5$ | $y = \frac{1}{2}x$ | $y = 3x - 7$ |
| $y = 3x$ | $y = 2x - 6$ | $y = -2x$ | $y = 3x + 5$ |
| $y = 4x$ | $y = 2x$ | $y = x$ | $y = 3x$ |
| $y = \frac{1}{2}x$ | $y = 2x + 1$ | $y = x + 6$ | $y = 3x - 1$ |

It may also be helpful to students to use the table function of the calculator after typing in each equation. Students can determine how each set of numbers is changing and verify that the ratios remain the same.

As a final activity, give students the equation $y = 4x - 9$ and have them create a table of values for the equation. Using the calculators, they can type in the linear function and check their work.

## The Function Game: Can You Guess the Secret?

Copy the input and output numbers for each secret given by your teacher. Write your guess for what is happening to the input number to create the output number for each table in the space below.

A.

| INPUT | OUTPUT |
|---|---|
| | |

My Guess:

D.

| INPUT | OUTPUT |
|---|---|
| | |

My Guess:

B.

| INPUT | OUTPUT |
|---|---|
| | |

My Guess:

E.

| INPUT | OUTPUT |
|---|---|
| | |

My Guess:

C.

| INPUT | OUTPUT |
|---|---|
| | |

My Guess:

F.

| INPUT | OUTPUT |
|---|---|
| | |

My Guess:

THE SECRET CALCULATORS

Numbers were entered into calculators. Try to find the ***secret*** and figure out what was done to the number put in the calculator (the input) to get the number that came out of the calculator (the output).

| input | output |
|---|---|

the secret in words:

______________________

______________________

the secret in symbols:

| input | output |
|---|---|

the secret in words:

______________________

______________________

the secret in symbols:

| input | output |
|---|---|

the secret in words:

______________________

______________________

the secret in symbols:

| input | output |
|---|---|

the secret in words:

______________________

______________________

the secret in symbols:

For this next task, you know the secret rule for the calculator. Can you find input and output numbers and write the rule in symbols? Try it! You can use the calculator to help you.

| input | output |
|---|---|
| | |

the secret in words:

*add 5 to a number to find a new number*

the secret in symbols:

| input | output |
|---|---|
| | |

the secret in words:

*multiply a number by 5 and add 1 to find a new number*

the secret in symbols:

| input | output |
|---|---|
| | |

the secret in words:

*decrease a number by 1 to find a new number*

the secret in symbols:

| input | output |
|---|---|
| | |

the secret in words:

*multiply a number by 3 and subtract 1 to find a new number*

the secret in symbols:

THE SECRET CALCULATORS ***Teacher Key***

Numbers were entered into calculators. Try to find the ***secret*** and figure out what was done to the number put in the calculator (the input) to get the number that came out of the calculator (the output).

input | output — the secret in words: — the secret in symbols:

***add 8 to an input number to make an output number*** — ***$y = x + 8$***

input | output — the secret in words: — the secret in symbols:

***double the input number and add 5 to make the output number*** — ***$y = 2x + 5$***

input | output — the secret in words: — the secret in symbols:

***subtract 2 from an input number to find an output number*** — ***$y = x - 2$***

input | output — the secret in words: — the secret in symbols:

***multiply the input by 2 and then add 6 for an output number*** — ***$y = 2x + 6$ or $y = 2(x + 3)$***

For this next task, you know the secret rule for the calculator. Can you find input and output numbers and write the rule in symbols? Try it! You can use the calculator to help you. ***Answers to the input output tables will vary.***

| input | output |
|---|---|
| | |

the secret in words: *add 5 to a number to find a new number*

the secret in symbols: $y = x + 5$

| input | output |
|---|---|
| | |

the secret in words: *multiply a number by 5 and add 1 to find a new number*

the secret in symbols: $y = 5x + 1$

| input | output |
|---|---|
| | |

the secret in words: *decrease a number by 1 to find a new number*

the secret in symbols: $y = x - 1$

| input | output |
|---|---|
| | |

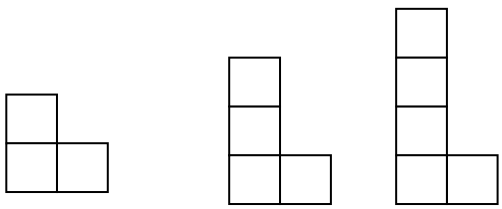
the secret in words: *multiply a number by 3 and subtract 1 to find a new number*

the secret in symbols: $y = 3x - 1$

# Tile Patterns                                    Activity 1

Directions: Use the square tiles to construct the next design in the pattern. Use two colors to show which part of the design remains the same each time and which part is changing. Discuss ideas with your team and then sketch the next two pictures. Work with your team to answer the questions below.

FIRST PATTERN DESIGN



Pattern one    Pattern two    Pattern three    Pattern four    Pattern five

1. How is the pattern  changing and how will the ninth pattern  look?

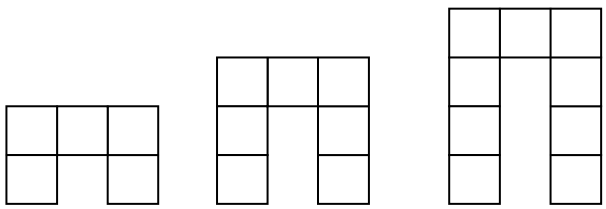
2. How would you explain to someone how to build any pattern number?

3. What part of the pattern is *constant* ?

4. What part of the pattern is *changing or variable?*

5. Write a rule using algebraic notation to describe how to build any pattern . Let ***n*** represent the number of the pattern and let ***a*** represent the amount of tiles. Test your rule with the patterns you have drawn.

SECOND PATTERN DESIGN



Pattern one    Pattern two    pattern three    pattern four    pattern five

1. How is the pattern changing and how will the ninth pattern look?

2. How would you explain to someone how to build any pattern number?

3. What part of the pattern is *constant* ?

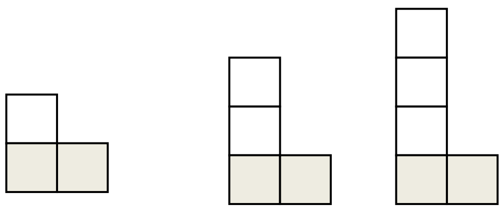
4. What part of the pattern is *changing or variable?*

5. Write a rule using algebraic notation to describe how to build any pattern . Let ***n*** represent the number of the pattern and let ***a*** represent the amount of tiles. Test your rule with the patterns you have drawn.

Tile Patterns ANSWER KEY Activity 1

Directions: Use the square tiles to construct the next design in the pattern. Use two colors to show which part of the design remains the same each time and which part is changing. Discuss ideas with your team and then sketch the next two pictures. Work with your team to answer the questions below.

FIRST PATTERN DESIGN ***accept reasonable responses***



Pattern one Pattern two Pattern three Pattern four Pattern five

1. How is the pattern changing and how will the ninth pattern look?
***Each new pattern has one more tile; the ninth will have eleven tiles: two on the bottom and nine stacked on top of the left bottom one***

2. How would you explain to someone how to build any pattern number?

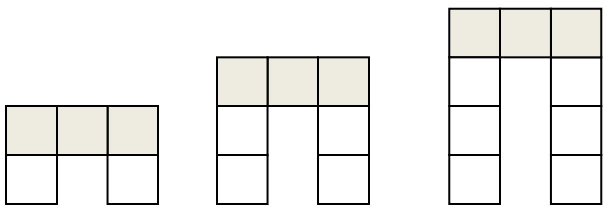
***The first tower is one more than the pattern number and then put one tile next to the first tower.***

3. What part of the pattern is *constant* ? ***the bottom two tiles are constant***

4. What part of the pattern is *changing or variable?* ***the number of tiles on top of the two on the bottom***

5. Write a rule using algebraic notation to describe how to build any pattern . Let $n$ represent the number of the pattern and let $a$ represent the amount of tiles. Test your rule with the patterns you have drawn. $a = n + 2$

SECOND PATTERN DESIGN ***accept reasonable responses***



Pattern one Pattern two pattern three pattern four pattern five

1. How is the pattern changing and how will the ninth pattern look?
***The "legs" of the bridge are growing; each new pattern has two more than the one before; the ninth pattern will have three across the top and each leg will be made of nine tiles for a total of 21 tiles.***

2. How would you explain to someone how to build any pattern number?
***Start at the top and place three tiles, add 2 legs that are equal to the number of the pattern. If the pattern number is four, you add eight tiles.***

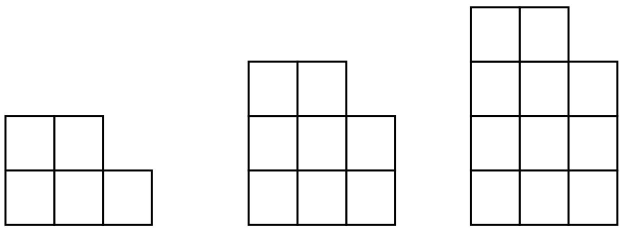
3. What part of the pattern is *constant*? ***the three tiles across the top***

4. What part of the pattern is *changing or variable?* ***The number of tiles on each "leg"***

5. Write a rule using algebraic notation to describe how to build any pattern. Let $n$ represent the number of the pattern and let $a$ represent the amount of tiles. Test your rule with the patterns you have drawn. $a = 2n + 3$

Tile Patterns Activity 2

Study the first three tile patterns and then sketch a picture of the fourth and fifth patterns.



pattern one pattern two pattern three pattern four pattern five

Complete the following table with information from the five designs above:

| Pattern number (n) | Variable amount | Amount constant | Total amount (a) |
|---|---|---|---|
| 1 | | | |
| 2 | | | |
| 3 | | | |
| 4 | | | |
| 5 | | | |

What do you notice about the relationship between the pattern number and the variable or changing amount?

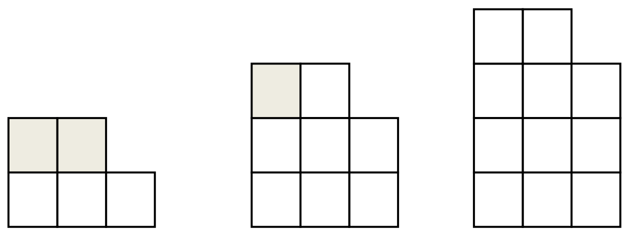
Describe how you could predict the total amount of tiles in the tenth pattern.

Using ***n*** for the pattern number and ***a*** for the total amount of tiles, write a rule that you could use to find the total amount of tiles in any pattern number. Compare your answer with classmates.

Tile Patterns **_ANSWER KEY_** Activity 2

Study the first three tile patterns and then sketch a picture of the fourth and fifth patterns.

**_ACCEPT REASONABLE RESPONSES_**



pattern one pattern two pattern three pattern four pattern five

Complete the following table with information from the five designs above:

| Pattern number (n) | Variable amount | Amount constant | Total amount (a) |
|---|---|---|---|
| 1 | **_3_** | **_2_** | **_5_** |
| 2 | **_6_** | **_2_** | **_8_** |
| 3 | **_9_** | **_2_** | **_11_** |
| 4 | **_12_** | **_2_** | **_14_** |
| 5 | **_15_** | **_2_** | **_17_** |

What do you notice about the relationship between the pattern number and the variable or changing amount?

**_The variable amount is three times the pattern number_**

Describe how you could predict the total amount of tiles in the tenth pattern.

**_Multiply three times ten and then add the constant of 2 for a total of 32_**

Using ***n*** for the pattern number and ***a*** for the total amount of tiles, write a rule that you could use to find the total amount of tiles in any pattern number. Compare your answer with classmates.

**_a = 3n + 2_**