Plumbing and Pipe-Fitting Challenges

Students often wonder when they will use the math they learn in school. These activities answer that question as it relates to measuring, working with fractions and decimals, and basic geometry. Plumbers need to have a good working knowledge of these and other math concepts to solve problems that arise on the job. To enter any of the building trades, an apprentice must score well on basic math tests. A solid understanding of basic math opens the door to many financially and personally rewarding careers.

Levels 5th through 8th grade

Topics Measuring lengths, Converting fractions to decimals, Adding and subtracting fractions and mixed numbers with like and unlike denominators, Using benchmark fractions, Reducing fractions, Approximating irrational numbers to the nearest eighth, Pythagorean Theorem, Angles

Goals

- Students will experience real world problems that motivate to measure and compute accurately
- Students will improve their fluency with problem solving involving fractions and decimals
- Students will practice using the Pythagorean Theorem

Bloom's Taxonomy Knowledge, Comprehension, Application, Analysis

Prerequisite Knowledge Students should have been exposed to fraction equivalence, addition and subtraction of mixed numbers, converting fractions to decimals, and the Pythagorean Theorem.

Preparation Time 1 to 2 hours

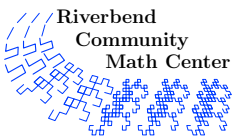
Activity Time 60 to 90 minutes each day – the activity is designed to last at least two days. The activity can be extended into a sequence of classes.

Materials for 30 Students

- 30 Handouts
- Blank paper (30 sheets)
- Saran wrap for wrapping pipes
- 6 team bags for fittings and small pipes
- 30 Pencils
- 30 Rulers
- 15 Tape measures
- 15 Pipe cutters
- 28 5-foot length PVC pipes (3/4" diameter)
- 300 90 degree elbow connectors (3/4")
- 300 45 degree connectors (3/4")
- 300 T connectors (3/4")
- 30 striaght connectors (3/4")
- Digital camera
- First Aid Kit
- Posterboard templates for checking models
- 12 to 24 pipe holders with clamps (Day 2)
- 6 Screwdrivers (Day 2)
- 6 Screw-in clamps (Day 2)
- 6 Wooden frames (Day 2)
- 1 Roll of masking tape (Day 2)
- 6 Styrofoam or cardboard boxes (Day 2)
- 6 Calculators (Day 2)

Primary Sources Many thanks to Robin Forsythe and other members of Plumbers & Pipefitters Local Union 172 for help with developing and field testing these activities

hello@riverbendmath.org
http://riverbendmath.org
(574) 339-9111



Day 1 Overview – 60 to 90 minutes

Introductions – 5 minutes

Introduce the plumbers to students (if they are present) and ask them to talk about their careers.

Fraction Counting Warm-ups – 5 minutes

If the students need to review reading a ruler, ask them to complete the fraction counting warm-ups.

Adding and Subtracting Fractions on the Ruler – 10 minutes

Even if the students know how to add and subtract mixed numbers on paper, it is useful to ask them to learn to use their rulers to complete the addition and subtraction problems in this section. The purpose of this part of the activity is to improve computational flexibility and number sense relating to fractions.

Determining Take-Offs – 10 minutes

Ask students to look at the various fittings. Explain that plumbers name angles in terms of the deviation from a straight line of flow rather than in terms of the interior angle. Thus, a "45 degree fitting" has an interior angle of 135 degrees.

Discuss the concept of a take-off as a group and ask students to discover take-off values for various fittings. Come back as a group to compare the values obtained. The following are the expected take-offs for 3/4" PVC pipe: 90 degree fittings have a take-off of 1/2", 45 degree fittings have a take-off of 3/8", and T degree fittings have a take-off of 5/8". However, actual values may differ by a small amount because we are not filing or lubricating the edges of the pipe ends as a plumber would. Students should use the values they find rather than the official take-offs.

Blueprint Challenges – 25 to 45 minutes

- Discuss safe use of tools. – 5 minutes
- Ask students to figure take-offs for first blueprint and create a supply list. – 5 minutes
- Obtain supplies. Measure and cut pipes to proper lengths and assemble model. – 10 minutes
- To prevent leaks, a plumber must create a model that is accurate to the nearest 1/8 of an inch. Once model is complete, students should check the model against the appropriate poster board template outline. (Trace a model with correct measurements onto the posterboard to create the template.)
- If time allows, repeat these steps for two other blueprints. – 10 minutes

Create Their Own Models and Blueprints – 10 to 30 minutes

If there is extra time, allow students to create their own models and blueprints.

Cleanup – 5 minutes

Day 2: Offset and Travel – 50 to 90 minutes

Finishing Work From Day 1

Teams that still need to finish blueprint challenges from Day 1 can continue to work on those.

Fraction to Decimal Conversion Chart – 10 minutes

The purpose of this part of the activity is to make it easier for students to approximate irrational numbers obtained from the Pythagorean Theorem to the nearest 1/8 of an inch. Making the chart also reviews concepts of fraction equivalence.

Offset, Advance, and Travel – 20 minutes

To complete this challenge, students should use the Pythagorean Theorem together with the fact that a 45-45-90 triangle is an isosceles triangle. This means that the Offset and the Advance are always equal. Students will need to remember to subtract the take-offs for the two 45 degree fittings.

Computing Travel – 15 minutes

In this part of the activity, two pipe clamps holding pipes with an offset are taped with masking tape to the work surface. Students use their tools to determine the offset. They can either slide the pipes or use straight fittings to adjust the Advance to the correct value.

Pipe-Running Challenge – 40 minutes

If there is enough time, create a pipe-running course for each team. A pipe clamp marks the starting line for the run of pipe. The students must make the pipe go around the wooden frame, but the pipe should pass close enough to be clamped to the outside of the frame. The pipe should continue to another point where it T's, sending one pipe to a styrofoam or cardboard box representing a furnace needing coolant, and sending the other pipe farther along the table and then down to hit a taped X on the floor which represents the drain.

Ideally, students should plan their design before acquiring the pipes. They should cut the pipe away from the work site and bring them back to their course only after cutting. This will encourage students to measure and think ahead. They tend to hold the pipe up and cut it in place without this requirement. Remind students that real pipes are often too large to be held in place while being cut, so plumbers usually need to measure and plan carefully.

Clean-up – 5 minutes

Hints for Teachers

Angles and Fittings

45° fittings actually have an internal angle os 135° and an external angle of 225°. So why are they called 45° fittings? Plumbers think in terms of water flow. A 45° fitting diverts the flow of water 45° from the straight path it would otherwise follow. Note that 45° and 135° are *supplementary angles*.

Types of Length Measurements

Plumbers use the following terms to describe the kind of measurement needed in a given situation. An *end-to-end* measurement is the distance from one end of a pipe to another. An *end-to-center* measurement is used when there is a fitting on only one end of a pipe. The end-to-center measurement is the distance from the point on a fitting where the two center lines meet to the end of a pipe. A *center-to-center* measurement is used when both ends of a pipe have a fitting. The center-to-center measurement is the distance between the intersections of the center lines on each fitting.

Pipe Lengths for the Blueprint Challenges

Remember to subtract off the appropriate take-off values from the end-to-center or center-to-center measurements on the blueprint. If students are using different values for the take-offs they will obtain different end-to-end lengths.

Challenge 1: The end-to-end lengths for the component pipes are $2\frac{5}{8}$", $5\frac{1}{4}$", $6\frac{3}{8}$", and 6".

Challenge 2: The end-to-end lengths for the component pipes are $3\frac{1}{4}$" (2 pipes), $3\frac{3}{8}$" (4 pipes), and $5\frac{1}{4}$" (2 pipes).

Challenge 3: The end-to end lengths for the component pipes are 4" (2 pipes), $4\frac{3}{8}$" (4 pipes), and $3\frac{3}{4}$" (1 pipe).

Geometric Challenge

If there is additional time on either day, this geometric challenge can be posed verbally.

Build an example of each of the regular polygons that can be made using the fittings availabe to us. Explain why these are the only possible regular polygons that can be constructed. What other convex polygons are possible? (Remember that regular polygons must have identical angles and identical edge lengths. Convex polygons do not have any angles that are concave or "dented in".

The only *regular polygons* that can be built using our fittings are squares and octagons. If you use only 90° fittings and pipes of equal length, then you will end up with a square. If you use only 45° fittings and pipes of equal length, then you will build an octagon. Convex polygons that are possible include irregular hexagons and rectangles.

Offset, Advance, and Travel

There are a number of difficulties that students may face in completing the construction challenges in this section. Students need to remember to account for take-off when cutting their pipes in both pipe challenges.

When using the 45° fittings, students will need to use the Pythagorean Theorem to find the center-to-center measurement for the travel. Once they find this length as a decimal, they will need to convert the decimal to the nearest eighth of an inch. One way to do this is to find decimal equivalents for $\frac{1}{8}$, $\frac{2}{8}$, $\frac{3}{8}$, and so forth and then determine which of these benchmark fractions is closest to the decimal obtained for the travel. After these calculations students must remember to account for the take-off.

There are at least two advantages to using 45° fittings instead of 90° fittings when connecting two parallel pipes. One advantage is that less pipe is used overall and so the cost of materials is less. Another advantage is that 45° fittings disrupt the flow of water less than 90° fittings and are less likely to become clogged.

The advance and the offset are the same because the pictured triangle must be a 45°, 45°, 90° triangle. This is an isosceles right triangle. The two legs of this triangle must be the same length and so the advance and the offset are identical.

If the offset is 6", the advance must also be 6". Use the Pythagorean Theorem to find that the travel must be approximately 8.485". Since $\frac{3}{8} = 0.375$ and $\frac{4}{8} = \frac{1}{2} = 0.5 = 0.500$, we can see that the travel rounded to the nearest eighth is $8\frac{1}{2}$".

Since we are using 45° fittings at the ends of the travel pipe, we need to take off $\frac{3}{8}$" for each of these fittings. This gives an end-to-end length of $7\frac{3}{4}$.

The plumbers' rule of thumb is to multiply the offset by 1.414 to find the travel. We can see that this works for any length if we let x stand for the length of the offset. The length of the advance is also x. If we let c stand for the length of the travel, then we have

$$\begin{aligned} c^2 &= x^2 + x^2 \\ c^2 &= 2x^2 \\ c &= \sqrt{2x^2} \\ c &= \sqrt{2}x \\ c &\approx 1.414x \end{aligned}$$

If the travel is equal to 10", we can find the offset (which is equal to the advance) by dividing by 1.414. This gives 7.072". Since $\frac{1}{8} = 0.125$, $7\frac{1}{8}$ is this measurement to the nearest eighth.

Related Indiana State Standards

- 5.2.2: Add and subtract fractions (including mixed numbers) with different denominators.
- 5.4.1: Measure, identify, and draw angles, perpendicular and parallel lines, rectangles, triangles, and circles by using appropriate tools (e.g., ruler, compass, protractor, appropriate technology, media tools).
- 5.4.7: Understand that 90°, 180°, 270°, and 360° are associated with quarter, half, three-quarters, and full turns, respectively.
- 5.7.3: Apply strategies and results from simpler problems to solve more complex problems.
- 5.7.7: Make precise calculations and check the validity of the results in the context of the problem.
- 5.7.9: Note the method of finding the solution and show a conceptual understanding of the method by solving similar problems.
- 6.1.4: Convert between any two representations of numbers (fractions, decimals, and percents) without the use of a calculator.
- 6.1.5: Recognize decimal equivalents for commonly used fractions without the use of a calculator.
- 6.1.7: Find the least common multiple and the greatest common factor of whole numbers. Use them to solve problems with fractions (e.g., to find a common denominator to add two fractions or to find the reduced form for a fraction).
- 6.2.5: Solve problems involving addition, subtraction, multiplication, and division of positive fractions and explain why a particular operation was used for a given situation.
- 6.2.10: Use mental arithmetic to add or subtract simple fractions and decimals.
- 6.7.4: Apply strategies and results from simpler problems to solve more complex problems.
- 6.7.7: Select and apply appropriate methods for estimating results of rational-number computations.
- 6.7.9: Make precise calculations and check the validity of the results in the context of the problem.
- 6.7.11: Note the method of finding the solution and show a conceptual understanding of the method by solving similar problems.
- 7.1.2: Compare and order rational and common irrational numbers and place them on a number line.
- 7.2.5: Use mental arithmetic to compute with simple fractions, decimals, and powers.
- 7.4.3: Know and understand the Pythagorean Theorem and use it to find the length of the missing side of a right triangle and the lengths of other line segments. Use direct measurement to test conjectures about triangles.
- 7.7.4: Apply strategies and results from simpler problems to solve more complex problems.
- 7.7.8: Select and apply appropriate methods for estimating results of rational-number computations.
- 7.7.10: Make precise calculations and check the validity of the results in the context of the problem.
- 7.7.12: Note the method of finding the solution and show a conceptual understanding of the method by solving similar problems.
- 8.1.7: Calculate and find approximations of square roots.
- 8.2.1: Add, subtract, multiply, and divide rational numbers (integers, fractions, and terminating decimals) in multi-step problems.
- 8.2.4: Use mental arithmetic to compute with common fractions, decimals, powers, and percents.
- 8.4.5: Use the Pythagorean Theorem and its converse to solve problems in two and three dimensions.
- 8.7.4: Apply strategies and results from simpler problems to solve more complex problems.
- 8.7.10: Make precise calculations and check the validity of the results in the context of the problem.
- 8.7.12: Note the method of finding the solution and show a conceptual understanding of the method by solving similar problems.